FINAL TECHNICAL REPORT

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Research Title:

Correlation of the Earth's Rotation Rate and the Secular Change of the Geomagnetic Field

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I. INTRODUCTION

The main objective of the nine-month research grant was to apply the power spectral density analysis using the maximum entropy method (MEM) to the geomagnetic dipole field for the years 1901 to 1969. Another objective was to apply the spherical harmonic analysis to the geomagnetic data prior to 1900. The first objective was accomplished. In addition, the application of the MEM spectral analysis was extended to include the length of day fluctuations. From the early phase of the work on the spherical harmonic analysis of the pre-1900 geomagnetic data, it was concluded that the annual mean magnetic data covering a span of 14 or more years is needed to fit a spherical harmonic expansion of the magnetic potential to the 7th order and 7th degree. Since the IBM 370-125 computer at Florida Institute of Technology is too slow to be practical for this work, no further progress was made. The necessity of gaining access to the IBM 360-75 computer at Goddard Space Flight Center was mentioned in the semi-annual status report [Jin, 1975].

The application of MEM spectral analysis to the dipole field of the earth and the length of day fluctuations indicate that both spectra have common relative maxima at 0.015 cycles/year and its harmonics. The method of analysis and results are given in the following sections of this report.

2. RESEARCH ACCOMPLISHED AND METHODS OF ANALYSIS

2.1 Power Spectral Analysis of the Geomagnetic Dipole Field

Power spectral density analysis using the maximum entropy method was originated by Burg [1967, 1968]. The method has been successful in estimating the power spectral density when the periods are comparable to the original data length [Lacoss, 1971; Ulrych, 1972]. For a discrete stochastic process, the MEM spectral density at a frequency f is given by:

$$P(f) = \frac{P_{M+1}}{f_N \left| 1 + \sum_{k=1}^{M} a_{Mk} \exp\left[-i2\pi kf(\Delta t)\right] \right|^2} \tag{1}$$

where:

- P_{M+1} is the mean output power of the (M + 1) - point prediction - error - filter that is used to whiten the time series;
- a_{Mk}'s are the prediction-error-filter coefficients that are determined from the data;
- Δt is the uniform sampling rate;
- f_N is the Nyquist frequency, and equals to $1/(2\Delta t)$;
- and f is the frequency between $-f_N$ and f_N for which the power spectral density is to be determined.

The prediction-error-filter coefficients, a_{Mk}, in Equation (1) are calculated from the data using the Burg algorithm [Ulrych and Bishop, 1975], without requiring either a zero or periodic extension of the data.

The values of the geomagnetic dipole field from 1901 to 1969 used for the MEM power spectral analysis were determined from the Gauss-Schmidt coefficients resulting from a spherical harmonic analysis of the annual mean magnetic data at various geomagnetic observatories [Jin, 1973]. The digitized annual mean values of geomagnetic data were provided by the Geomagnetic Division, U.S. Coast and

Geodetic Survey. It is essentially the same as those given in "The Annual Mean Value of Geomagnetic Elements Since 1900" submitted in 1965 by the working group on the analysis of the geomagnetic field to the International Association of Geomagnetism and Aeronomy. Additional data up to 1969 are included on the digitized tape; however, the data for 1967, 1968 and 1969 are not complete. For the spherical harmonic analysis, the geomagnetic field near the earth's surface is assumed to be mainly of internal origin and can be represented by the negative gradient of a magnetic potential function. The magnetic potential function V is expanded in terms of the spherical harmonic function, i.e.

$$V = a \sum_{n=1}^{\infty} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) P_n^m(\theta)$$

where:

r, θ, and ϕ are the geocentric radius, colatitude, and longitude;

a is the mean earth radius;

g_n^m and h_n^m are the Gauss-Schmidt coefficients;

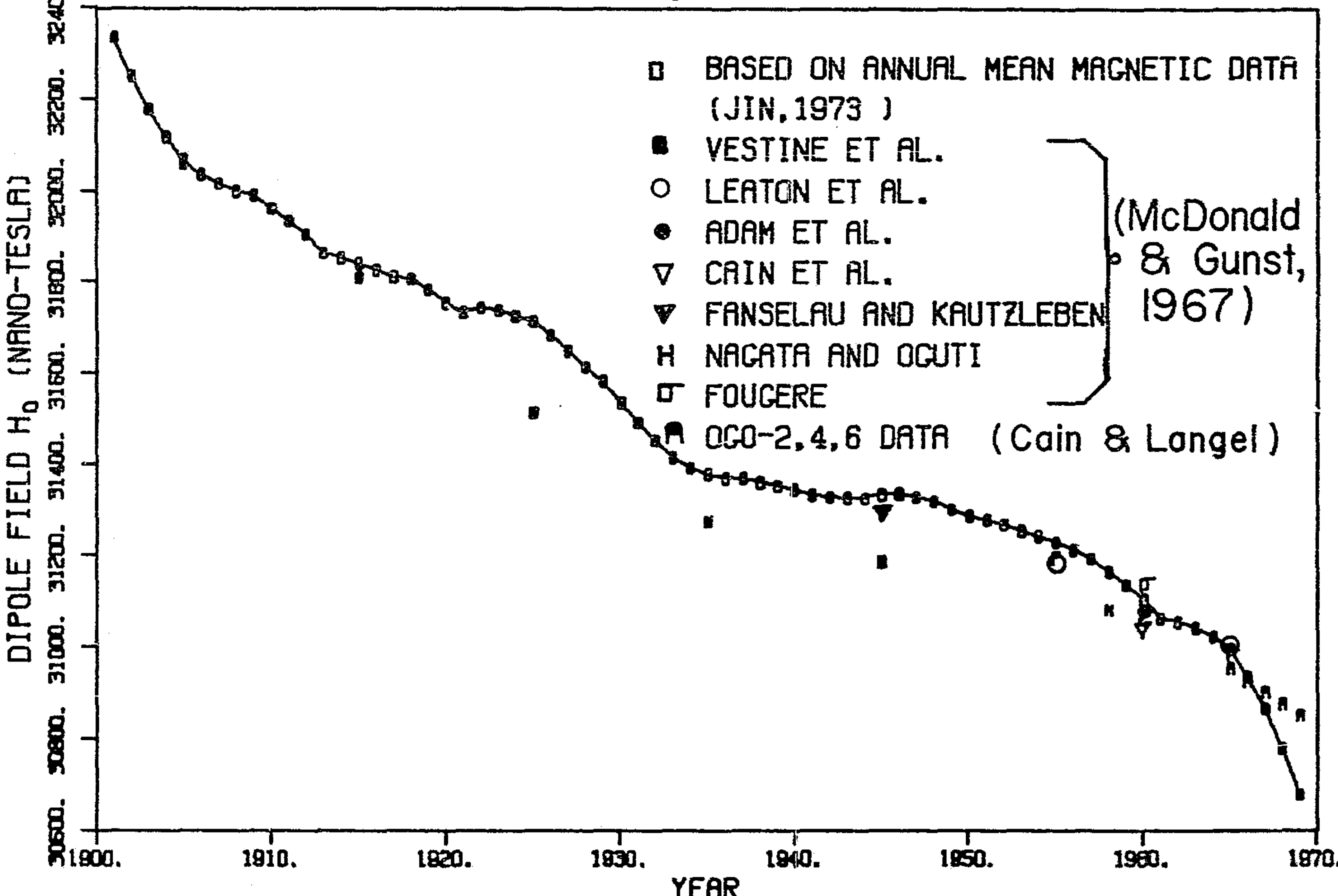
and $P_n^m(\theta)$ are the Schmidt partially normalized Associated Legendre functions.

The spherical harmonic expansion was truncated to the 7th order and 7th degree. Earth oblateness was also taken into account. The least squares method was used in fitting the observatory data to obtain the Gauss-Schmidt coefficients. The method has been well documented by Cain, et al [1967]. In order to obtain the global secular changes of the geomagnetic field, nine year data spans were used for analysis and the Gauss-Schmidt coefficients were assumed to be parabolic functions of time. The values of these coefficients together with their first and second time derivatives were obtained for the mid-point of the nine-year span. Different nine-year spans with mid-points separated by four years for the period 1905 to 1965 were analyzed. The dipole moment M is given by $H_0 a^3$ where H_0 is the dipole field at the magnetic equator and is given by:

$$H_0 = \left[(g_1^0)^2 + (h_1^0)^2 + (h_1^1)^2\right]^{1/2} \qquad (2)$$

The resulting dipole field H_0 from 1901 to 1969 is shown in Figure 1. In applying

FIGURE 1: Dipole Field of the Earth



the MEM power spectral analysis to the dipole field, the number of prediction-error-filter coefficients were varied from 9 to 68. The number of spectral estimates is chosen to be 300 separated by 1.67×10^{-3} cycles/year. The existence of the relative maxima in the dipole field spectra at 0.015 cycles/year and its higher harmonics were found. The existence of the fundamental frequency and its harmonics are evident when the number of the prediction-error-filter coefficients exceeds 80% of the number of points in the time series of the dipole field. If the number of the prediction-error-filter coefficients is less than 80% of the record length, only some of the harmonics of the 0.015 cycles/year will appear. A typical power spectra of the geomagnetic dipole field is shown in Figure 2. Since the time series of the geomagnetic dipole field is nonstationary, and the criterion in choosing the length of the prediction-error-filter coefficients is lacking, we decided to test the 80% requirement. For that purpose, a time series composed of sine waves with 0.015 cycles/year and four harmonics was generated. In addition, a linear function derived from a linear fit to the geomagnetic dipole field H_0 was added to the sine waves. The MEM power spectral analysis was then applied to this series by varying the number of prediction-error-coefficients used. It was reassuring that the spectral line at the fundamental frequency cannot be found unless the number of prediction-error-filter coefficients again exceeds 80% of the record length. The MEM power spectral analysis was also applied to the time rate of change of the dipole field. The time derivative $\dot{H}_0$ was determined from the H_0 of nine consecutive years by solving for B in the parabolic function:

$$H_0(t) = A + Bt + Ct^2 \quad , \qquad t = -4 \text{ to } +4 \tag{3}$$

where A, B, and C are constants.

The time series of $\dot{H}_0$ from 1905 to 1965 is given in Figure 3. Its power spectra is shown in Figure 4. Again the spectral peaks at 0.015 cycles/year and its harmonics are evident.

2.2 Power Spectral Analysis of the Length of Day Fluctuations

It is known that the earth's rotation rate is not constant in that the observed positions of the sun, moon, and planets based on a rotating earth (Universal Time)

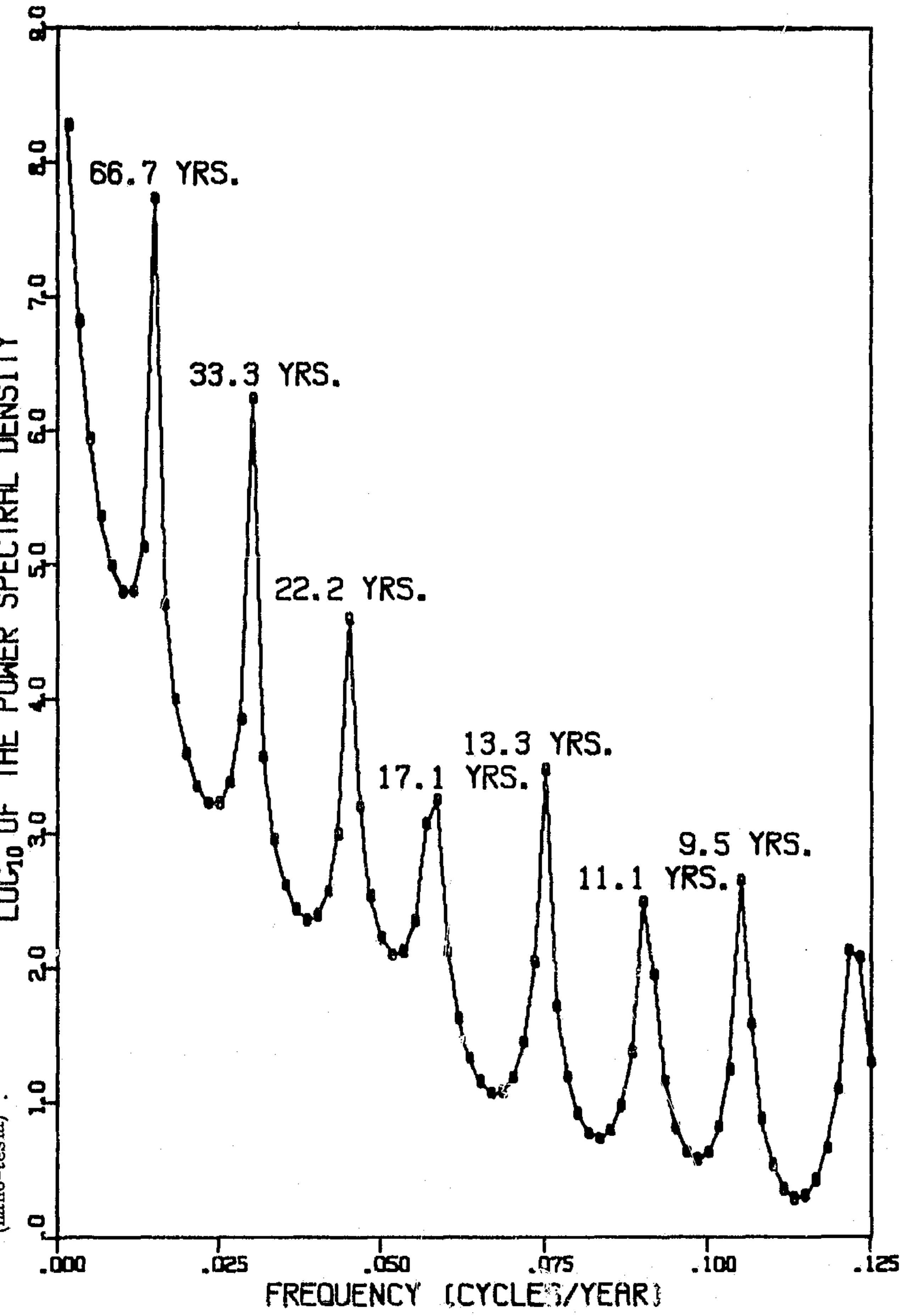


FIGURE 2: Power Spectra of the Geomagnetic Dipole Field for the Period 1901 to 1969. The number of prediction-error-filter coefficients used is 68, and the number of spectral estimates is 300. The unit for power spectral density is (nano-tesla)^2.

FIGURE 3: The Time Rate of Change of Geomagnetic Dipole Field

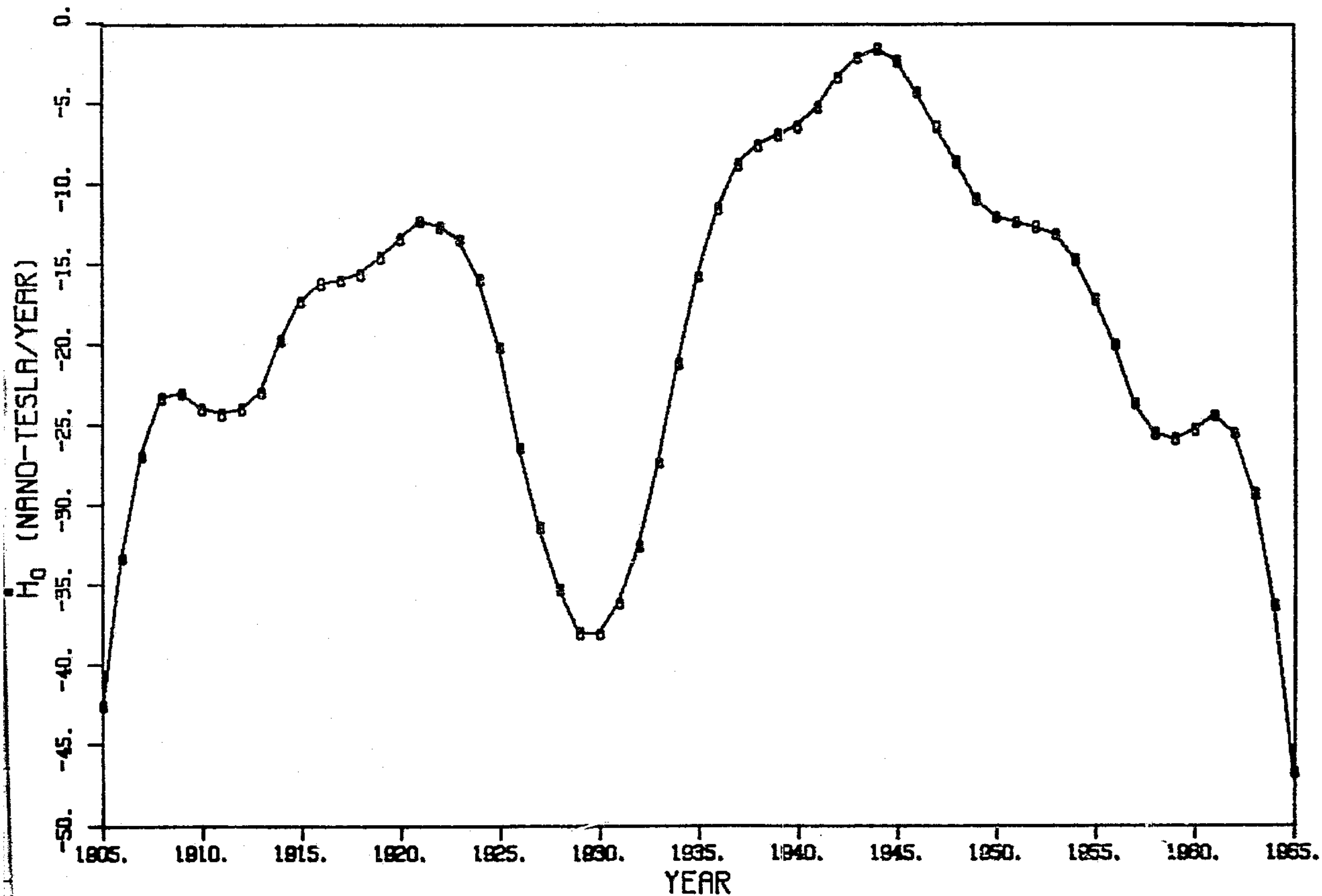
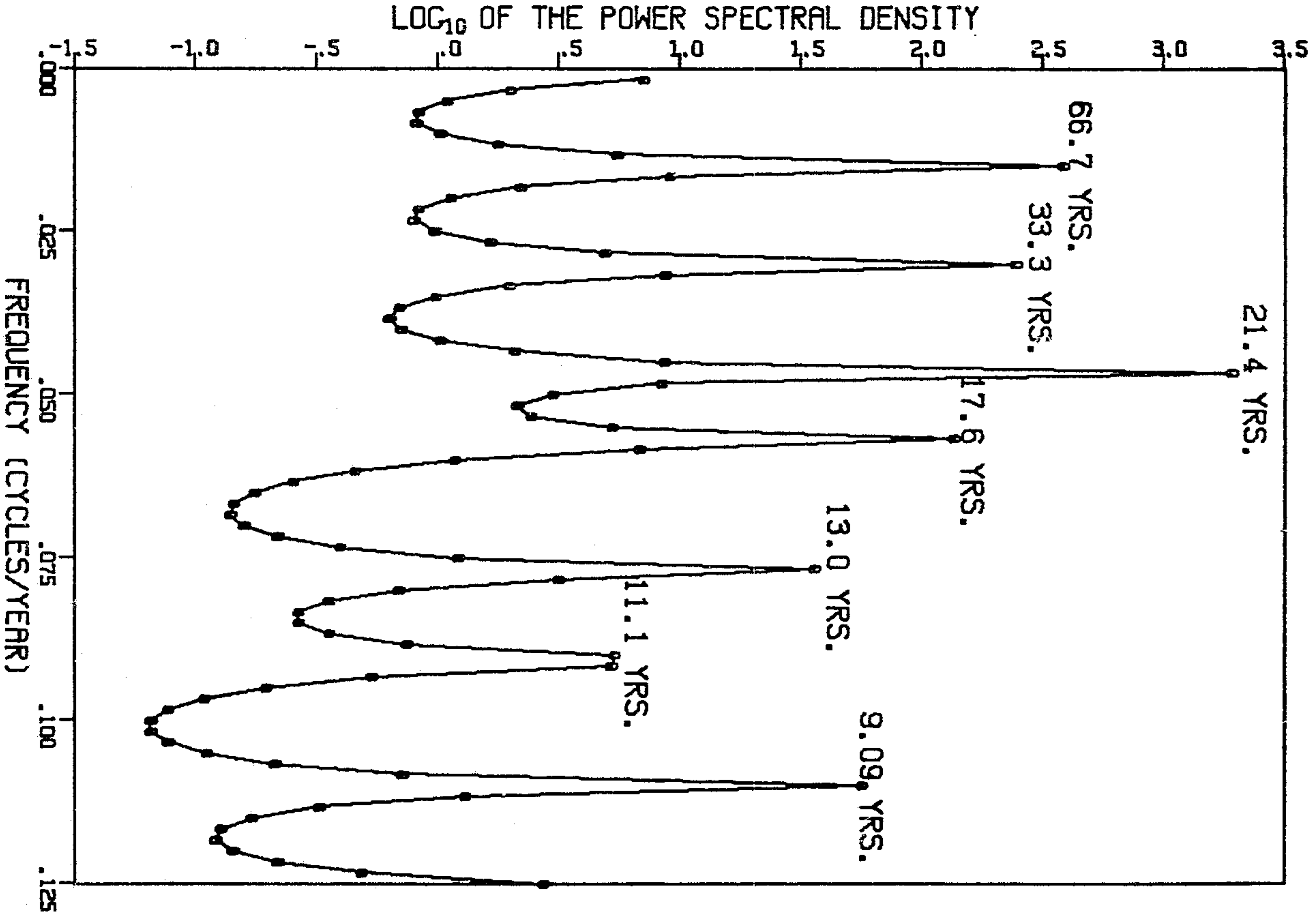


FIGURE 4: Power Spectra of the Time Rate of Change of the Geomagnetic Dipole Field for the Period 1905 to 1965. The number of prediction-error-filter coefficients used is 61, and the number of spectral estimates is 300. The unit for power spectral density is (nano-tesla year)^2.



are different from those calculated from Newtonian mechanics based on a uniform time (Ephemeris Time). The difference ΔT between the Ephemeris Time and the Universal Time prior to 1950 are given by Brouwer [1952]. The values for ΔT after 1950 can be found in the American Ephemeris and Nautical Almanac. The time discrepancy ΔT is given by:

$$\begin{aligned}\Delta T &= \text{Ephemeris Time} - \text{Universal Time} \\ &= 24.349\ \Delta L \text{ sec.}\end{aligned}$$

where 24.349 is the time required for the sun's mean longitude to increase 1 sec. of arc, and ΔL is the difference between the observed and the theoretical longitudes in seconds of arc. The difference ΔL is given by $\Delta L = a_o + b_o t + f_o$ where a_o, b_o are constants related to particular origin of time, January 1, 1900, and the rate of change of the Ephemeris Time; and f_o is the longitude discrepancy due to the fluctuations and the secular changes in the earth's rotation rate [Munk and MacDonald, 1960]. The so called l.o.d. fluctuations is proportional to the time derivative of ΔT, i.e.,

$$\text{l.o.d fluctuations} = 2.738 \times 10^{-3} \times d(\Delta T)/dt \text{ sec. of Ephemeris Time}$$

where 2.738×10^{-3} is the reciprocal of the number of mean solar days in a year, and $d(\Delta T)/dt$ is the annual rate of change of ΔT in seconds. For the MEM spectral analysis, the values of ΔT from 1861 to 1948 were taken from Table 8a of Brouwer's article [1952], and those from 1948 to 1965 were taken from the American Ephemeris and Nautical Almanac. The derivative of ΔT from 1865 to 1961 were then obtained from the nine point formula, similar to Equation (3). The time series of ΔT and its time derivative are given separately in Figures 5 and 6.

The MEM power spectral analysis was applied to the time series of $d(\Delta T)/dt$ which is proportional to the l.o.d. fluctuations. The number of prediction-error-filter coefficients used were 20, 40, 60 and 80, and the number of spectral estimates was 300 separated by 1.67×10^{-3} cycles/year. The spectral peaks at 0.015 cycles/year and its harmonics were found when the number of prediction-error-filter coefficients are 40, 60 and 80. A lower frequency peak at 8.23×10^{-3} cycles/year was also found when the number of prediction-error-filter coefficients is 80. However,

FIGURE 3: Time Discrepancy Curve

ΔT = Ephemeris Time – Universal Time

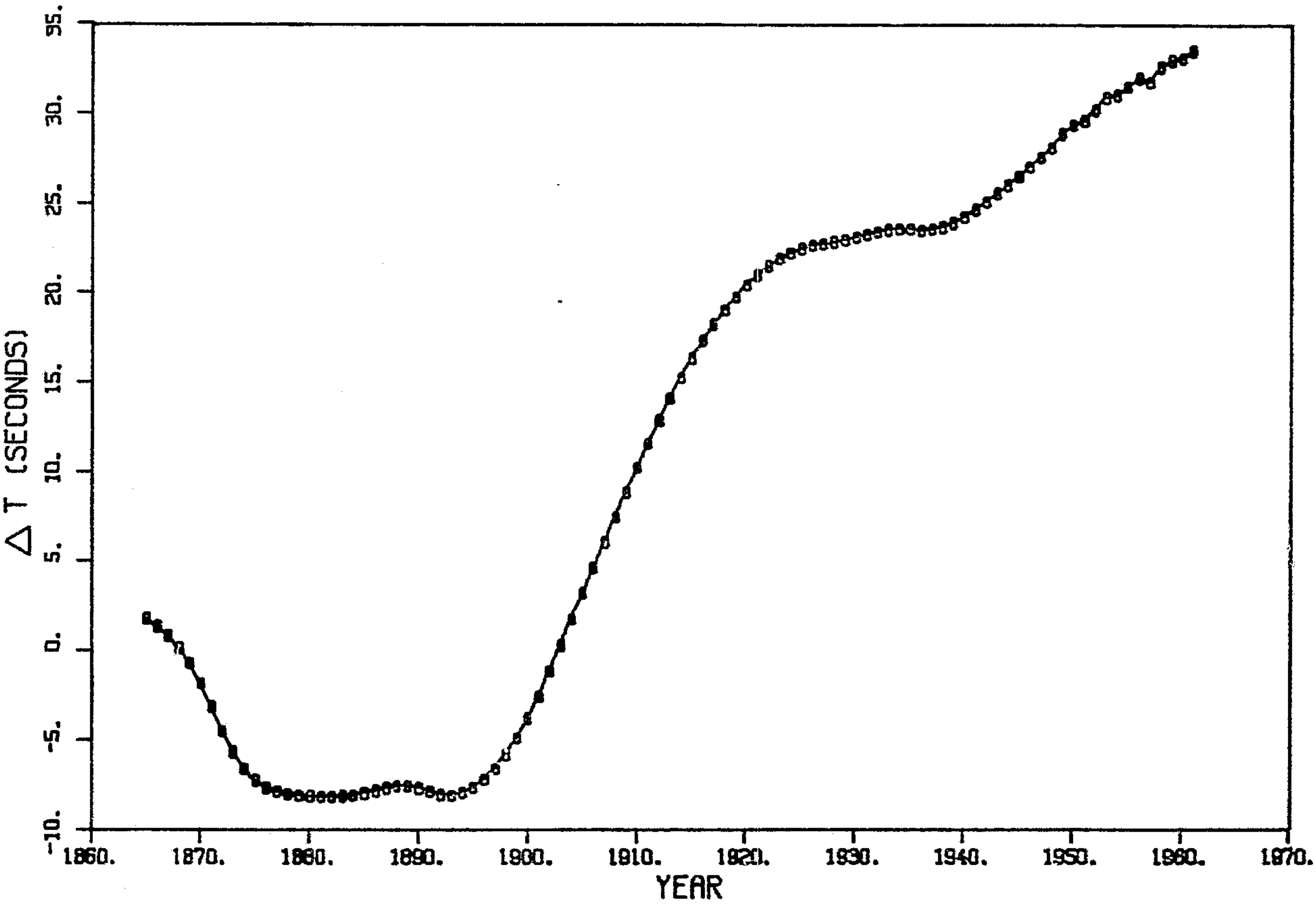
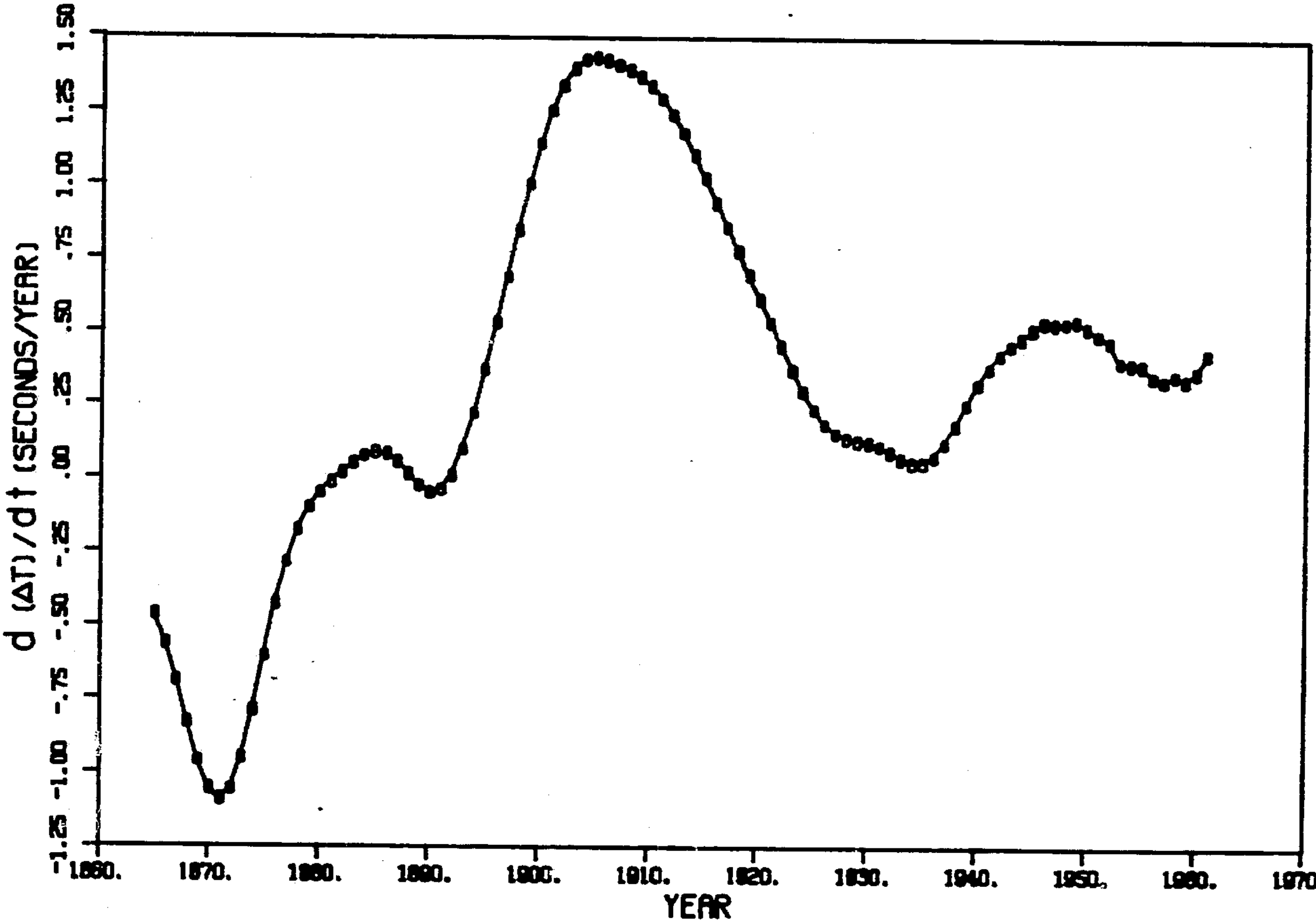


FIGURE 6: The Time Rate of Change of ΔT. The l.o.d. fluctuation equals $2.738 \times 10^{-3} \times d(\Delta T)/dT$ sec. of Ephemeris Time.

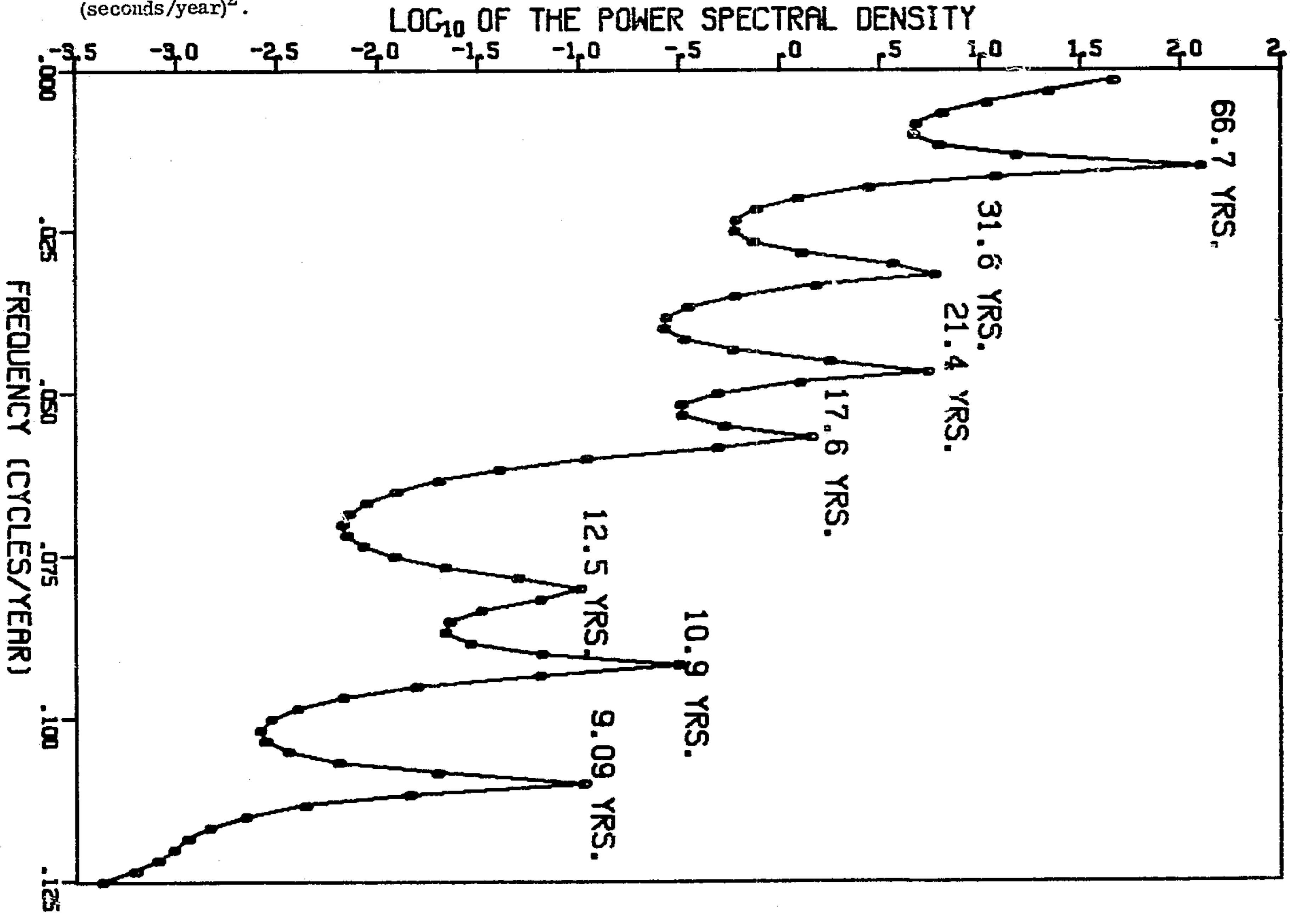


this 120-year spectral line is considered spurious. A typical power spectra of the time derivative of ΔT is given in Figure 7.

3. RESULTS AND CONCLUSIONS

The application of MEM spectral analysis to the dipole field and its derivative from 1901 to 1969 uncovered the existance of spectral peaks at 0.015 cycles per year and its harmonics. They correspond to 66, 33, 22, 17, 13, 11, 9-year spectral lines. Currie [1973a; 1973b] applied the MEM spectral analysis to separate time series of mean yearly magnetic data at 28 geomagnetic observatories and reported the existence of the 11-year solar cycle, the 22-year solar magnetic cycle, and an approximately 60-year spectral line. Our spectral analysis on the time series of the geomagnetic dipole field confirms the existence of the ≈60 year spectral line, however, the fact that we observed clearly the peaks at higher harmonics of the fundamental frequency 0.015 cycles/year, the 22-year and 11-year spectral lines cannot be attributed to the solar magnetic cycle and the solar cycle unambiguously. It is interesting to note that a 11-year cycle not attributed to the solar cycle was found in the horizontal component of the geomagnetic field [Rivin, 1974]. Furthermore, it is remarkable that the same spectral lines were also observed in the spectrum of the l.o.d. fluctuations from 1865 to 1961. The common spectral lines in the l.o.d. fluctuations and the geomagnetic dipole field variations suggests that either the cause of both phenomena is the same such as the magneto-hydrodynamic oscillation in the fluid core of the earth or one is the cause of the other. Whichever is the case, the fundamental frequency 0.015 cycles/year and its higher harmonics should be related to the motion inside the fluid core of the earth for the past one hundred years. As for the phase relation between the l.o.d. fluctuations and the geomagnetic dipole field variations, this can only be resolved through a cross correlation study of the two phenomena.

FIGURE 7: Power Spectra of the Time Rate of Change of ΔT for the Period 1865 to 1965. The numb[illegible] of prediction-error-filter coefficients used is 60, and the number of spectral estimates is 300. The unit for power spectral density is $(\text{seconds/year})^2$.



4. ADDENDUM

These research results will be reported in an article "Spectral Line Similarity in the Geomagnetic Dipole Field Variations and Length of Day Fluctuations" to be submitted to the Journal of Geophysical Research for publication. The abstract is given in the Appendix. The preprints of the paper will be provided to NASA at the time of submission for publication.

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APPENDIX

Spectral Line Similarity in the Geomagnetic Dipole Field Variations and Length of Day Fluctuations

ABSTRACT

Power spectral density analysis using Burg's maximum entropy method was applied to the geomagnetic dipole field and its rate of change for the years 1901 to 1969. Both spectra indicate relative maxima at 0.015 cycles/year and its harmonics. These maxima correspond approxmately to 66, 33, 22, 17, 13, 11, and 9-year spectral lines. The application of the same analysis techniques to the length-of-day (l.o.d.) fluctuations for the period 1865 to 1961 reveal similar spectral characteristics. Since we observe clearly peaks at higher harmonics of the fundamental frequency, the 22-year and 11-year lines cannot be attributed in an unambiguous manner to the solar magnetic cycle and the solar cycle. We suggest that this similarity in the l.o.d. fluctuations and the dipole field variations is related to the motion within the earth's fluid core during the past one hundred years.